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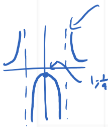
Chapter 1 Review Quadratic Functions math 10/11 Honours

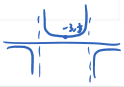
1. Given that $f(x)=\frac{3}{2}x+12$ and $g(x)=3(x-2)^2-4$, indicate the domain and range for each of the following

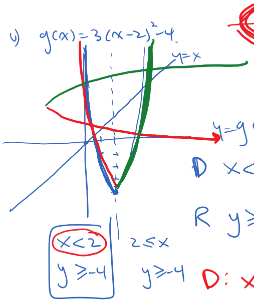
	Domain:	Range:
i) $y=f(x)$	$x \in R$	$y \in R$
ii) $y=f^{-1}(x)$	$x \in R$	$y \in R$
iii) $y=\lvert f(x)\rvert$	$x \in R$	$y \geq 0$
iv) $y=\frac{1}{f(x)}$	$x \in R,\ x \neq -8$	$y \in R,\ y \neq 0$
v) $y=g^{-1}(x)$ $y=g(x)$	$x \geq -4$ →	$y \geq 2$ or ← Top $y < 2$ ← Bottom
vi) $y=\lvert g(x)\rvert$	$x \in R$	$y \geq 0$
vii) $y=\frac{1}{g(x)}$	$x \in R,\ x \neq 2 \pm \frac{2\sqrt{3}}{2}$	$y \geq 0,\ y \leq -\frac{1}{4}$

2. Given each reciprocal function, find the coordinates of the invariant points, equation of the asymptotes, and the domain & range:

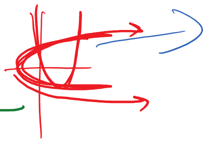


	i) $y=\frac{1}{(x+1)^2-9}$	ii) $y=\frac{1}{-2(x+3)^2+8}$ ← squared!?
	Inver $(x+1)^2-9=1$ $(x+1)^2=10$ $x=-1\pm\sqrt{10}$ $(x+1)^2-9=-1$ $(x+1)^2=8$ $x=-1\pm2\sqrt{2}$ $1, \frac{1}{9}$ Asy $(x+1)^2-9=0$ $x+1=\pm3$ $x=2/-4$	Inver $-2(x+3)^2+8=1$ $(x+3)^2=\frac{7}{2}$ $x=-3\pm\frac{\sqrt{14}}{2}$ $-2(x+3)^2+8=-1$ $(x+3)^2=\frac{9}{2}$ $x=-3\pm\frac{3\sqrt{2}}{2}$ Asy $+2(x+3)^2=+8$ $(x+3)=\pm2$ $x=-1/-5$
Invariant Points:	$(-1+\sqrt{10},1)(-1-\sqrt{10},1)(-1\pm2\sqrt{2},-1)$ ← ±	$(-3\pm\frac{\sqrt{14}}{2},1)(-3\pm\frac{3\sqrt{2}}{2},-1)$
Equation of Asymptotes	$x=2,\ x=-4$	$x=-1,\ x=-5$
Domain:	$x\neq2,\ x\neq-4$	$x\neq-1,\ x\neq-5$
Range:	$y>0,\ y\leq-\frac{1}{9}$	$y\geq\frac{1}{8},\ y<0$





← $g(x)$

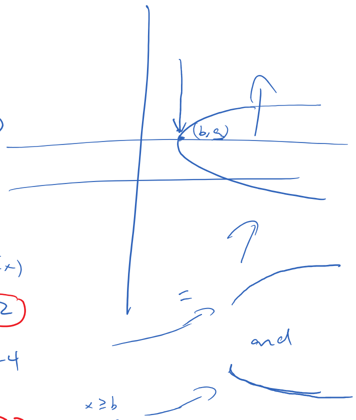


$y=g^{-1}(x)$

D $x<2$ → R: $y<2$

R $y\geq-4$ → D $x\geq-4$

D: $x\geq2$ → R: $y\geq2$



$y\geq2$ or $y\leq2$

	iii) $y=\frac{1}{x^2+2x+4}=\frac{1}{(x+1)^2+3}$	ii) $y=\frac{1}{3x^2+12}$ → $\frac{1}{3(x-0)^2+12}$
	Invar $(x+1)^2+3=1$ X $=-1$ X Asy No x intercepts	Invar? $3x^2+12=1$ X -1 X $0, \frac{1}{12}$
Invariant Points:	NO. ← $(-1,\frac{1}{3})$	NO
Equation of Asymptotes	NO.	NO (no x-ints)
Domain:	$x\in R$	$x\in R$
Range:	$0<y\leq\frac{1}{3}$	$0<y\leq\frac{1}{12}$

3. Given that $f(x)=3(x-4)^2-10$, solve the equation: $f(x)=f^{-1}(x)$

① they intersect on $y=x$!

② $x=3(x-4)^2-10$

$x=3x^2-24x+38$

$0=3x^2-25x+38$

③ $x=\frac{25\pm\sqrt{625-4(3)(38)}}{6}$

$x=\frac{25\pm13}{6}$

$x=\frac{19}{3}/2$

4. Solve by Completing the Square

a) $5x^2-30x+8=0$

$5(x^2-6x)+8=0$

$5(x^2-6x+9-9)+8=0$

$5(x-3)^2-37=0$

$(x-3)^2=\frac{37}{5}$

$x-3=\pm\sqrt{\frac{37}{5}}$

$x=3\pm\frac{\sqrt{185}}{5}$

b) $-\frac{1}{3}x^2+4x-5=0$

$-\frac{1}{3}(x^2-12x)-5=0$

$-\frac{1}{3}(x^2-12x+36)+12-5=0$

$-\frac{1}{3}(x-6)^2=-7$

$(x-6)^2=21$

$x=6\pm\sqrt{21}$

c) $6x^2+30x+5=0$

$(x^2+5x+\frac{25}{4})-\frac{25}{4}(6)+5=0$

$6(x+\frac{5}{2})^2=\frac{65}{2}$

$(x+\frac{5}{2})^2=\frac{65}{12}$

$x=-\frac{5}{2}\pm\frac{\sqrt{195}}{6}$

d) $-\frac{1}{2}x^2-\frac{9}{2}x+5=0$

$-\frac{1}{2}(x^2+9x+\frac{81}{4})+\frac{81}{8}+5=0$

$-\frac{1}{2}(x+\frac{9}{2})^2=\frac{121}{8}$

$x+\frac{9}{2}=\pm\frac{11}{2}$

$x=1/-10$

5. For what values of *k* does each equation have two different real roots?

a) $x^2 + kx + 9 = 0$

$k^2 - 4(1)(9) > 0$

$k^2 > 36$

$\underline{k < -6 \quad k > 6}$

b) $3x^2 + kx + 27 = 0$

$k^2 - 4(3)(27) > 0$

$k^2 > 324$

$\underline{k < -18 \quad k > 18}$

6. For what values of *m* does each equation have two equal real roots?

a) $4x^2 + mx + 9 = 0$

$m^2 - 4(4)(9) = 0$

$m^2 = 144$

$\underline{m = \pm 12}$

b) $(2m-1)x^2 - 8x + 6 = 0$

$64 - 4(6)(2m-1) = 0$

$64 - 48m + 24 = 0$

$88 = 48m$

$\underline{m = 11/6}$

7. For what values of *n* does each equation have no real roots?

a) $5x^2 + mx + 20 = 0$

$m^2 - 4(5)(20) < 0$

$m^2 < 400$

$\underline{-20 < m < 20}$

b) $nx^2 - 5x + n = 0$

$25 - 4(n)(n) < 0$

$\frac{25}{4} < n^2$

$-\frac{5}{2} \quad \frac{5}{2}$

$n < -\frac{5}{2}$ or $\frac{5}{2} < n$

8. A ball is thrown into the air from the balcony of a condo and falls to the ground. The height h meters of the ball relative to the ground t seconds after being thrown is given by $h = -5t^2 + 18t + 20$. When will the ball reach 28 meters?

$-5t^2 + 18t + 20 = 28$

$-5t^2 + 18t - 8 = 0$

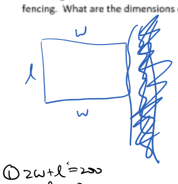
$t = \frac{-18 \pm \sqrt{18^2 - 4(5)(8)}}{-10}$

$t = \frac{-18 \pm \sqrt{164}}{-10}$

$\boxed{t_1 = \frac{-18 + \sqrt{164}}{-10} \approx 0.519}$ THIS IS WHEN THE BALL FIRST REACHES 28m.

$t_2 = \frac{-18 - \sqrt{164}}{-10} \approx 3.08s$ THIS IS WHEN THE BALL IS FALLING TO 28m

9. A rectangular lot is bordered on one side by a stream and on the other three sides by 200 meters of fencing. What are the dimensions of the lot if its area is 4350 m^2.



$2w + l = 200$

$lw = 4350$

$l = \frac{4350}{w}$

$2w + \frac{4350}{w} = 200$

$2w^2 - 200w + 4350 = 0$

$w = 50 \pm 5\sqrt{13}$

$(50 + 5\sqrt{13},\ 100 - 10\sqrt{13})$

$(50 - 5\sqrt{13},\ 100 + 10\sqrt{13})$

① $2w + l = 200$

$l = -2w + 200$

② $A = l \times w$

$4350 = (-2w + 200)w$

$0 = -2w^2 + 200w - 4350$

$0 = w^2 - 100w + 2175$

$w = \frac{100 \pm \sqrt{100^2 - 4(2175)}}{2}$

$w = \frac{100 \pm \sqrt{1300}}{2} = \frac{100 \pm 10\sqrt{13}}{2}$

1ST CASE

$\therefore w = 50 + 5\sqrt{3}$

$l = 100 - 10\sqrt{3}$

2ND CASE

$w = 50 - 5\sqrt{3}$

$l = 100 + 10\sqrt{3}$

10. The second number is 4 more than 3 times the first number and their product are 480. Find the numbers.

$y = 3x + 4$

$xy = 480$

$x(3x+4) = 480$

$3x^2 + 4x - 480 = 0$

$x = \frac{-4 \pm 76}{6}$

$= \cancel{36}$ or $-\frac{40}{3}$

$\cancel{y = \frac{40}{3}}$

$y = -36$

11. A metal wire, 40 cm long, is cut in two and each piece bent to form a square. If the sum of their areas is 58 cm^2, how long is each piece of wire?

① THE TWO PIECES OF WIRE ARE x & $40-x$ IN LENGTH.

② EACH PIECE IS BENT TO MAKE A SQUARE:

x $\quad$ $40-x$

$\frac{x}{4}$ $\quad$ $\frac{40-x}{4}$

$A_1 = \frac{x^2}{16}$ $\quad$ $A_2 = \left(\frac{40-x}{4}\right)^2$

③ $A_1 + A_2 = 58$

$\frac{x^2}{16} + \frac{(40-x)^2}{16} = 58$

$x^2 + (1600 - 80x + x^2) = 58 \times 16$

④ SOLVE FOR 'x'

$2x^2 - 80x + 1600 = 928$

$2x^2 - 80x + 672 = 0$

$x^2 - 40x + 336 = 0$

$(x-28)(x-12) = 0$

$x = 28 \quad x = 12$

⑤ CHECK.

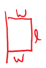
THE TWO PIECES ARE 28cm & 12cm

$A_1 = \left(\frac{28}{4}\right)^2 = 49$ $\quad$ $A_2 = \left(\frac{12}{4}\right)^2$, $A_2 = 9$

$\therefore x_1 = 28cm$

$x_2 = 12cm$

12. A family plans to fence in a rectangular patio area behind their house. They have 200 feet of fence to use. One side of the rectangle is the back of the house. What should be the dimensions of the rectangular region if they want to make the patio area enclosed as large as possible



① $2w + l = 200$

$l = -2w + 200$

② $A = l \times w$

$A = -2w^2 + 200w$

$= -2(w^2 - 100w)$

$= -2(w^2 - 100w + 2500 - 2500)$

$= -2(w - 50)^2 + 5000$

$\therefore w = 50m$

$l = 100m$

MAX AREA $= 5000m^2$

13. Solve the equation: $\frac{1}{3x-4} = |3x-4|$

① TWO CASES.

CASE #1

$\frac{1}{3x-4} = -(3x-4)$

$1 = -(3x-4)^2$

$-1 = (3x-4)^2$ NO SOL'N

CASE #2

$\frac{1}{3x-4} = 3x-4$

$1 = (3x-4)^2$

$\pm 1 = 3x - 4.$

$1 = 3x - 4$

$\boxed{\frac{5}{3} = x}$

$-1 = 3x - 4$

$3 = 3x$

$\boxed{1 = x}$

② CHECK FOR EXTRANEOUS ROOT

$\frac{1}{(3)\frac{5}{3} - 4} = \left|3\left(\frac{5}{3}\right) - 4\right|$

$1 = |1|$

$\boxed{x = 5/3 \text{ GOOD}}$

$\frac{1}{3(1)-4} = |3(1) - 4|$

$\frac{1}{-1} \neq -1$

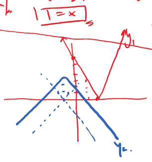
$\therefore x = 1$ ✗ IS AN EXTRANEOUS ROOT

14. Solve: $|x+2| + |-3x+6| = 3$

① BEST WAY TO DO THIS QUESTION IS BY GRAPHING IT OUT.

$|x+2| + |-3x+6| = 3.$

$\underbrace{|-3x+6|}_{Y_1} = \underbrace{-|x+2| + 3}_{Y_2}.$



SINCE THE TWO GRAPHS DON'T INTERSECT, NO INTERSECTION $\rightarrow$ NO SOL'N